# Joint Face Representation Adaptation and Clustering in Videos: Supplementary Material

Zhanpeng Zhang<sup>1</sup>, Ping Luo<sup>2,1</sup>, Chen Change Loy<sup>1,2</sup>, and Xiaoou Tang<sup>1,2</sup>

 Dept. of Information Engineering, The Chinese University of Hong Kong
Shenzhen Key Lab of Comp. Vis. & Pat. Rec., Shenzhen Institutes of Advanced Technology, CAS, China

This supplementary material presents more clustering results, and mathematical details for the character label inference in MRF.

# 1 More Clustering Results

Fig. 1 and 2 show more clusters generated by our method in the Accio [1] and BF0502 dataset [2].

#### 2 Details on Character Label Inference in MRF

### 2.1 The computation of pairwise term $\Psi(\cdot)$

As mentioned in Sec. 3.1 of the paper, given the face representation  $\mathbf{x}_i$  and  $\mathbf{x}_j$ , we define the pairwise term  $\Psi(\cdot)$  for the character label  $y_i$  and  $y_j$  by

$$\Psi(y_i, y_j) = \exp\left\{\alpha v(\mathbf{x}_i, \mathbf{x}_j) \cdot \left(\mathbf{1}(y_i, y_j) - \mathbf{1}(v(\mathbf{x}_i, \mathbf{x}_j) > 0)\right)\right\},\tag{1}$$

where  $\mathbf{1}(\cdot)$  is an indicator function and  $\alpha$  is a trade-off coefficient between unary term and pairwise term. This coefficient  $\alpha$  will be updated as stated in the following Sec. 2.2. Here we give the details on the computation of face pair relation  $v(\mathbf{x}_i, \mathbf{x}_j) \in V$ , which includes two steps:

- 1. Compute a normalized affinity matrix  $V^a$  based on the distance between  $\mathbf{x}_i$  and  $\mathbf{x}_i$ .
- 2. Propagate the initial pairwise constraints  $C_0^3$  by the affinity matrix  $V^a$  and obtain the pair relation V.

For the first step, to compute the normalized affinity matrix  $V^a$ , we follow the method used in spectral clustering [3]. Firstly, we compute an affinity matrix A by  $A_{ij} = \exp(-d^2(x_i, x_j)/\sigma_i\sigma_j)$  if  $x_j$  is within the h-nearest neighbors of  $x_i$ , otherwise we set  $A_{ij} = 0$ . We set h = 10 here as [4] (In fact, h is not a sensitive parameter. Usually  $h \in [5, 100]$  produces similar clustering accuracy

As stated in the paper, we obtain  $C_0$  by assuming all the face images in the same track have the same identity, *i.e.*,  $c(I_i, I_j) = 1$ . For faces appearing in the same frame of the video, their identities should be exclusive, *i.e.*,  $c(I_i, I_j) = -1$ . For other face pairs we have  $c(I_i, I_j) = 0$ .



**Fig. 1.** Example results generated by the proposed method on the Accio (*Harry Potter*) dataset. Every two rows denote a cluster.



**Fig. 2.** Example results generated by the proposed method on the "Buffy the Vampire Slayer" dataset [2]. Every row denote a cluster.

within 3% fluctuation). The term  $d(x_i, x_j)$  is the L2-distance between  $x_i$  and  $x_j$ , and  $\sigma_i$  is the local scaling factor with  $\sigma_i = d(x_i, x_m)$ , where  $x_m$  is the m-th nearest neighbor of  $x_i$ . We set m = 7 as in [3]. Then the normalized affinity

matrix is obtained by  $V^a = D^{-\frac{1}{2}}AD^{-\frac{1}{2}}$ , where D is a diagonal matrix with  $D_{ii} = \sum_{j=1}^{n} A_{ij}$ .

For the second step, we use the constraint propagation method in [5] to compute the pair relation V by:

$$V = (1 - \lambda)^2 (1 - \lambda V^a)^{-1} W (1 - \lambda V^a)^{-1}, \tag{2}$$

where W is a matrix form of the pairwise constraints  $C_0$  (i.e.,  $W_{ij} = c(I_i, I_j)$ ).  $\lambda$  controls the propagation degree ( $\lambda = 0.5$  in implementation). To this end, we can have V and compute the pairwise term  $\Psi(\cdot)$ .

#### 2.2 MRF optimization process

Given the face representation  $\mathbf{X}$ , we infer character  $\mathbf{Y}$  by maximizing the joint probability  $p(\mathbf{X}, \mathbf{Y})$  with the simulated field algorithm [6, 4]. The algorithm is as follows: (1) Initialization step: we initialize  $\mathbf{Y}$  by running K-means clustering on  $\mathbf{X}$ . In this case, for the Gaussian of each cluster  $\ell$ , we obtain the the initial mean  $\mu_{\ell}$  and covariance matrix  $\Sigma_{\ell}$ . We also set the initial trade-off coefficient  $\alpha = 0$  in Eqn. (1). For simplicity, we denote the model parameter  $\Omega = \{\mu_{\ell}, \Sigma_{\ell}, \alpha\}$  in the following text. (2) After the initialization step, we infer  $\mathbf{Y}$  and update  $\Omega$  by repeating the following two steps in each iteration q:

- 1. Simulate a new inferred  $\widetilde{\mathbf{Y}}^q$  given the face representation  $\mathbf{X}$  and current model parameter  $\Omega^q$ .
- 2. Given  $\widetilde{\mathbf{Y}}^q$ , update  $\Omega^q$  to maximize the log-likelihood of  $p(\mathbf{X}, \mathbf{Y})$  by EM algorithm.

For the first step, we aim to obtain a new  $\widetilde{\mathbf{Y}}^q$  given  $\mathbf{X}$  and  $\Omega^q$ . A natural way is to infer from the posterior:

$$p(\mathbf{Y}|\mathbf{X}, \Omega^q) = \frac{p(\mathbf{X}|\mathbf{Y}, \Omega^q)p(\mathbf{Y}|\Omega^q)}{p(\mathbf{X}|\Omega^q)}.$$
 (3)

However the computation of the term  $p(\mathbf{Y}|\Omega^q)$  involves the interaction of each  $y_i$  and its neighborhood. Thus, it is intractable. Here we employ the mean field-like approximation [6] for  $p(\mathbf{Y}|\Omega^q)$ , in which we assume each  $y_i$  is independent, and we set the value of its neighborhood  $\mathcal{N}_i$  constant when we compute  $p(y_i)$ . In this case, we have

$$p(\mathbf{Y}|\Omega^q) = \prod_i p(y_i|\mathbb{Y}_{\mathcal{N}_i}, \Omega^q) = \prod_i \frac{p(y_i, \mathbb{Y}_{\mathcal{N}_i}, \Omega^q)}{\sum_{y_i=\ell}^K p(y_i, \mathbb{Y}_{\mathcal{N}_i}, \Omega^q)}.$$
 (4)

where we denote the value of  $y_i$ 's neighborhood as  $\mathbb{Y}_{\mathcal{N}_i}$ . For example, we can reuse the value in the previous iteration q-1 (i.e.,  $\mathbb{Y}_{\mathcal{N}_i} = \widetilde{\mathbf{Y}}_{\mathcal{N}_i}^{(q-1)}$ ). Since  $p(y_i, \mathbb{Y}_{\mathcal{N}_i}, \Omega^q) = \frac{1}{Z} \prod_{j \in \mathcal{N}_i} \Psi(y_i, y_j)$ , the partition function Z can be eliminated

in Eqn. (4) and we can compute  $p(y_i|\mathbb{Y}_{\mathcal{N}_i},\Omega^q)$ . Combining Eqn. (3), Eqn. (4) and the mean field-like approximation, we have

$$p(\mathbf{Y}|\mathbf{X}, \Omega^q) = \prod_{i} p(y_i|\mathbb{Y}_{\mathcal{N}_i}, \mathbf{x}_i, \Omega^q) = \prod_{i} \frac{\Phi(\mathbf{x}_i|y_i, \Omega^q)p(y_i|\mathbb{Y}_{\mathcal{N}_i}, \Omega^q)}{\sum_{u=\ell}^K \Phi(\mathbf{x}_i|y_i, \Omega^q)p(y_i|\mathbb{Y}_{\mathcal{N}_i}, \Omega^q)}$$
(5)

Then the posterior  $p(y_i = \ell | \mathbb{Y}_{\mathcal{N}_i}, \mathbf{x}_i, \Omega^q)$  can be computed directly for each face i and cluster  $\ell$ . After that, we can simulate a new  $\widetilde{y_i}^q$  based on this posterior (*i.e.*, the probability of setting  $\widetilde{y_i}^q = \ell$  is proportional to  $p(y_i = \ell)$ ). Then we obtain  $\widetilde{\mathbf{Y}}^q = \{\widetilde{y_i}^q\}$ .

For the second step, we aim to maximize the log-likelihood of  $p(\mathbf{X}, \mathbf{Y})$  by updating the model parameter  $\Omega$  in an EM algorithm. We define

$$Q(\Omega|\Omega^q) = \mathbb{E}_{\mathbf{Y}|\mathbf{X},\Omega^{q-1}}(\log(p(\mathbf{X},\mathbf{Y}|\Omega^q))), \tag{6}$$

where  $\mathbb{E}$  denotes the expected value. So we have  $\Omega^{q+1} = \arg \max_{\Omega} \mathcal{Q}(\Omega | \Omega^q)$ .

Recall that (1)  $\Omega = \{\mu, \Sigma, \alpha\}$  and  $p(\mathbf{X}, \mathbf{Y}) = \frac{1}{Z} \prod_i \Phi(\mathbf{x}_i | y_i) \prod_i \prod_{j \in \mathcal{N}_i} \Psi(y_i, y_j)$ , (2)  $\mu$  and  $\Sigma$  are only related to the unary term  $\Phi$ , (3)  $\alpha$  is only related to the pairwise term  $\Psi$ . As in [6, 4], Eqn. (6) can be decomposed and we can update  $\{\mu, \Sigma\}$  and  $\alpha$  separately:

$$\mu^{q+1}, \Sigma^{q+1} = \arg\max_{\mu, \Sigma} \sum_{i} \sum_{y_i = \ell}^{K} p(y_i | \mathbb{Y}_{\mathcal{N}_i}, \mathbf{x}_i, \Omega^q) \log \Phi(\mathbf{x}_i | y_i \mu, \Sigma), \tag{7}$$

$$\alpha^{q+1} = \arg\max_{\alpha} \sum_{i} \sum_{y_i=\ell}^{K} p(y_i | \mathbb{Y}_{\mathcal{N}_i}, \mathbf{x}_i, \Omega^q) \log p(y_i | \mathbb{Y}_{\mathcal{N}_i}, \alpha).$$
 (8)

For Eqn. (7), since  $\Phi(\mathbf{x}_i|y_i\mu, \Sigma)$  is a Gaussian distribution and  $p(y_i|\mathbb{Y}_{\mathcal{N}_i}, \mathbf{x}_i, \Omega^q)$  is constant, we can have a closed form solution for  $\mu, \Sigma$ . As for Eqn. (8), we find a local optimal value for  $\alpha$ , by the local search method as in [4].

The optimization of the above two steps ends when the posterior  $p(y_i = \ell | \mathbb{Y}_{\mathcal{N}_i}, \mathbf{x}_i, \Omega^q)$  converged. The output character label  $\mathbf{Y}^* = \arg \max_{\mathbf{Y}} p(\mathbf{Y} | \mathbf{X}, \Omega)$ .

## References

- Ghaleb, E., Tapaswi, M., Al-Halah, Z., Ekenel, H.K., Stiefelhagen, R.: Accio: A data set for face track retrieval in movies across age. In: ACM International Conference on Multimedia Retrieval. (2015)
- 2. Everingham, M., Sivic, J., Zisserman, A.: Hello! My name is... buffy –automatic naming of characters in TV video. In: BMVC. (2006)
- Zelnik-Manor, L., Perona, P.: Self-tuning spectral clustering. In: NIPS. (2004) 1601–1608
- Wu, B., Zhang, Y., Hu, B.G., Ji, Q.: Constrained clustering and its application to face clustering in videos. In: CVPR. (2013)
- Lu, Z., Ip, H.H.: Constrained spectral clustering via exhaustive and efficient constraint propagation. In: ECCV. (2010)
- Celeux, G., Forbes, F., Peyrard, N.: EM procedures using mean field-like approximations for markov model-based image segmentation. Pattern recognition 36(1) (2003) 131–144